

PROOF OF FORMULA 3.314

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x} dx}{(e^{b/a} + e^{-x/a})^\nu} = ae^{b(\mu-\nu/a)} B(a\mu, \nu - a\mu)$$

Let $t = e^{-(x+b)/a}$ to obtain

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x} dx}{(e^{b/a} + e^{-x/a})^\nu} = \frac{1}{ae^{b(\mu-\nu/a)}} \int_0^{\infty} \frac{t^{\mu a-1} dt}{(1+t)^\nu}.$$

The result follows from the integral representation

$$B(\alpha, \beta) = \int_0^{\infty} \frac{t^{\alpha-1} dt}{(1+t)^{\alpha+\beta}}.$$