

PROOF OF FORMULA 3.316

$$\int_{-\infty}^{\infty} \frac{(1 + e^{-x})^{\nu} - 1}{(1 + e^{-x})^{\mu}} dx = \psi(\mu) - \psi(\mu - \nu)$$

Start with

$$\int_{-\infty}^{\infty} \frac{(1 + e^{-x})^{\nu} - 1}{(1 + e^{-x})^{\mu}} dx = \int_{-\infty}^{\infty} \left[\frac{1}{(1 + e^{-x})^{\mu - \nu}} - \frac{1}{(1 + e^{-x})^{\mu}} \right] dx.$$

Formula 3.317.2 states that

$$\int_{-\infty}^{\infty} \left[\frac{1}{(1 + e^{-x})^{\nu}} - \frac{1}{(1 + e^{-x})^{\mu}} \right] dx = \psi(\mu) - \psi(\nu).$$

The result is obtained by replacing ν by $\mu - \nu$.