PROOF OF FORMULA 3.324.2

$$\int_{-\infty}^{\infty} e^{-(x-b/x)^{2n}} \, dx = \frac{1}{n} \Gamma\left(\frac{1}{2n}\right)$$

Symmetry gives

$$\begin{aligned}\int_{-\infty}^{\infty} e^{-(x-b/x)^{2n}} \, dx &= 2\int_{0}^{\infty} e^{-(x-b/x)^{2n}} \, dx \\ &= 2\int_{0}^{\infty} e^{-(x-b/x)^{2n}} \, \frac{dx}{x^2}.\end{aligned}$$

The change of variables $t = b/x$ was employed in the last line.

Averaging these two expressions

$$\int_{-\infty}^{\infty} e^{-(x-b/x)^{2n}} \, dx = \int_{0}^{\infty} e^{-(x-b/x)^{2n}} \left(1 + \frac{b}{x^2}\right) \, dx.$$

The function $u = x - b/x$ is increasing for $b \geq 0$. Therefore, the change of variables $u = x - b/x$ gives

$$\int_{-\infty}^{\infty} e^{-(x-b/x)^{2n}} \, dx = \frac{1}{n} \int_{0}^{\infty} u^{1/2n-1} e^{-u} \, du.$$

The last integral is $\Gamma(1/2n)$.