PROOF OF FORMULA 3.329

$$\int_0^\infty \left[\frac{a\,\exp(-ce^{ax})}{1-e^{-ax}} - \frac{b\,\exp(-ce^{bx})}{1-e^{-bx}} \right] dx = \frac{\ln b - \ln a}{e^c}$$

Frullani's formula states that

$$\int_0^\infty \frac{F(at)-F(bt)}{t}\,dt = (F(\infty)-F(0))\ln\frac{a}{b}.$$

The present example is obtained by choosing $F(t) = t\,\exp\,(-ce{-}t)/(1-e^{-t})$ and observing that $F(\infty) = 0$ and $F(0) = e^{-c}$.