PROOF OF FORMULA 3.331.1

$$\int_0^\infty e^{-be^{-x}-ax}\,dx = b^{-a}\gamma(a,b)$$

The incomplete gamma function is defined by

$$\gamma(\alpha,x) = \int_0^x t^{\alpha-1}e^{-t}\,dt.$$

The change of variables $t = e^{-x}$ yields

$$\int_0^\infty e^{-be^{-x}-ax}\,dx = b^{-a}\int_0^b e^{-t}t^{a-1}dt.$$

That is the result.