PROOF OF FORMULA 3.331.2

$$\int_0^\infty e^{-be^x - ax}\, dx = b^a \Gamma(-a, b)$$

The *incomplete gamma function* is defined in 8.350.2 by

$$\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} dt.$$

The change of variables $t = e^x$ yields

$$\int_0^\infty e^{-be^x - ax}\, dx = \int_1^\infty e^{-bt} t^{a-1} dt.$$

Now let $s = bt$ to obtain the result.