PROOF OF FORMULA 3.331.4

$$\int_0^\infty (1-e^{-x})^{\nu-1} e^{-be^x-\mu x}\, dx = \Gamma(\nu) b^{(\mu-1)/2} e^{-b/2} W_{\frac{1-\mu-2\nu}{2},-\frac{\mu}{2}}(b)$$

The function in the answer is the *Whittaker function* defined by the integral representation

$$W_{a,b}(z) = \frac{z^{b+1/2}\, e^{-z/2}}{\Gamma(b-a+1/2)} \int_0^\infty e^{-zt}\, t^{b-a-1/2}\, (1+t)^{b+a-1/2}\, dt.$$

This appears in 9.222.1. The change of variables $t = e^{-x}$ gives the identity

$$\int_0^\infty (1-e^{-x})^{\nu-1} e^{-be^x-\mu x}\, dx = e^{-b} \int_0^\infty t^{\nu-1}(1+t)^{-\mu-\nu} e^{-bt}\, dt.$$

The result now follows by taking $a = -\mu/2 - \nu + 1/2$ and $b = -\mu/2$.