

**PROOF OF FORMULA 3.333.2**

$$\int_{-\infty}^{\infty} \frac{e^{-sx} dx}{\exp(e^{-x}) + 1} = \begin{cases} (1 - 2^{1-s})\Gamma(s)\zeta(s) & \text{if } s \neq 1 \\ \ln 2 & \text{if } s = 1 \end{cases}$$

Let  $t = e^{-x}$ . Then

$$\int_{-\infty}^{\infty} \frac{e^{-sx} dx}{\exp(e^{-x}) + 1} = \int_0^{\infty} \frac{t^{s-1} dt}{e^t + 1}.$$

For  $s \neq 1$  the value is computed in formula 3.411.3. For  $s = 1$ , the change of variables  $y = e^t + 1$  gives

$$\int_0^{\infty} \frac{dt}{e^t + 1} = \int_2^{\infty} \frac{dy}{y(y-1)}.$$

The partial fraction decomposition

$$\frac{1}{y(y-1)} = \frac{1}{y-1} - \frac{1}{y}$$

gives the result.