

PROOF OF FORMULA 3.351.2

$$\int_a^\infty x^n e^{-\mu x} dx = \frac{\Gamma(n+1, a\mu)}{\mu^{n+1}} = e^{-a\mu} \sum_{k=0}^{\infty} \frac{n!}{k!} \frac{a^k}{\mu^{n-k+1}}$$

Formula 2.321.2 states that

$$\int x^n e^{-\mu x} dx = -e^{-\mu x} \left(\sum_{k=0}^n \frac{k! \binom{n}{k}}{\mu^{k+1}} x^{n-k} \right).$$

Therefore

$$\int_a^\infty x^n e^{-\mu x} dx = e^{-\mu a} \sum_{k=0}^n \frac{n!}{k!} \frac{a^k}{\mu^{n-k+1}}.$$

The incomplete gamma function is defined in 8.350.2 by

$$\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1} e^{-t} dt,$$

and the change of variables $t = \mu x$ provides the evaluation

$$\int_a^\infty x^n e^{-\mu x} dx = \frac{\Gamma(n+1, a\mu)}{\mu^{n+1}}.$$