PROOF OF FORMULA 3.366.1

$$\int_0^{2a} \frac{(a-x)e^{-\mu x}\,dx}{\sqrt{2ax-x^2}} = \pi a e^{-a\mu} I_1(a\mu)$$

The integral representation

$$I_\nu(z) = \frac{z^\nu}{2^\nu \Gamma(\nu+1/2)\Gamma(1/2)} \int_{-1}^{1} e^{-zt}(1-t^2)^{\nu-1/2}\,dt$$

appears as 8.431.1. In particular

$$I_1(a) = \frac{1}{\pi}\int_{-1}^{1} \frac{te^{tz}\,dt}{\sqrt{1-t^2}}.$$

The change of variables $x = at$ gives

$$\int_0^{2a} \frac{(a-x)e^{-\mu x}\,dx}{\sqrt{2ax-x^2}} = \int_0^{2} \frac{(1-t)e^{-a\mu t}\,dt}{\sqrt{2t-t^2}}.$$

The result follows by completing the square to write

$$2t - t^2 = 1 - (t-1)^2$$

and using the change of variables $s = 1-t$.