

**PROOF OF FORMULA 3.366.2**

$$\int_0^\infty \frac{(x+b)e^{-\mu x} dx}{\sqrt{x^2+2bx}} = be^{b\mu} K_1(b\mu)$$

The integral representation for the Bessel function  $K_\nu$  appears in 8.432.3:

$$K_\nu(z) = \frac{z^\nu \sqrt{\pi}}{2^\nu \Gamma(\nu + 1/2)} \int_1^\infty e^{-tz} (t^2 - 1)^{\nu-1/2} dt.$$

In particular

$$K_1(z) = z \int_1^\infty e^{-zt} \sqrt{t^2 - 1} dt.$$

Complete the square to get

$$\int_0^\infty \frac{(x+b)e^{-\mu x} dx}{\sqrt{x^2+2bx}} = \int_0^\infty \frac{(x+b)e^{-\mu x} dx}{\sqrt{(x+b)^2 - b^2}}.$$

The change of variables  $s = x + b$  gives

$$\int_0^\infty \frac{(x+b)e^{-\mu x} dx}{\sqrt{(x+b)^2 - b^2}} = e^{b\mu} \int_b^\infty \frac{se^{-\mu s} ds}{\sqrt{s^2 - b^2}}.$$

Now let  $r = bs$  to obtain the result.