PROOF OF FORMULA 3.382.4

$$\int_0^\infty (x+b)^{-\nu} e^{-\mu x}\, dx = \frac{e^{b\mu}}{\mu^{\nu+1}} \Gamma(\nu+1, b\mu)$$

The *incomplete gamma function* appearing in the answer is defined by

$$\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1} e^{-t} dt.$$

This appears as 8.350.2.

The change of variables $t = x + b$ gives

$$\int_0^\infty (x+b)^{-\nu} e^{-\mu x}\, dx = e^{b\mu} \int_b^\infty t^\nu e^{-mut} dt.$$

The result is now obtained by letting $s = \mu t$.

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