PROOF OF FORMULA 3.411.12

$$\int_0^\infty \frac{xe^{-2nx}}{1+e^x}dx = \frac{\pi^2}{12} + \sum_{k=1}^{2n} \frac{(-1)^k}{k^2}$$

Formula 3.411.8 states that

$$\int_0^\infty \frac{x^{m-1}e^{-px}\,dx}{1+e^x} = (m-1)!\sum_{k=1}^\infty \frac{(-1)^{k-1}}{(k+p)^m}.$$

The special case $m=2$ and $p=2n$ yields

$$\int_0^\infty \frac{xe^{-2nx}\,dx}{1+e^x} = \sum_{k=1}^\infty \frac{(-1)^{k-1}}{(k+2n)^2}.$$

The series simplifies as follows

$$\sum_{k=1}^\infty \frac{(-1)^{k-1}}{(k+2n)^2} = \sum_{j=2n+1}^\infty \frac{(-1)^{j-1}}{j^2} = \sum_{j=1}^\infty \frac{(-1)^{j-1}}{j^2} - \sum_{j=1}^{2n} \frac{(-1)^{j-1}}{j^2}.$$

The result now follows from

$$\sum_{j=1}^\infty \frac{(-1)^{j-1}}{j^2} = \frac{\pi^2}{12}.$$