PROOF OF FORMULA 3.411.3

$$\int_0^\infty \frac{x^{\nu-1}\,dx}{e^{ax}+1} = \frac{1-2^{1-\nu}}{a^\nu}\Gamma(\nu)\zeta(\nu)$$

Let $t = ax$ to obtain

$$\int_0^\infty \frac{x^{\nu-1}\,dx}{e^{ax}+1} = a^{-\nu}\int_0^\infty \frac{t^{\nu-1}\,dt}{e^t+1}.$$

Integrate the expansion

$$\frac{1}{e^t+1} = \frac{e^{-t}}{e^{-t}+1} = \sum_{k=0}^\infty (-1)^k e^{-(k+1)t},$$

and let $s = (1+k)t$ to produce

$$\int_0^\infty \frac{x^{\nu-1}\,dx}{e^{ax}+1} = a^{-\nu}\sum_{k=0}^\infty \frac{(-1)^k}{(k+1)^\nu}\int_0^\infty s^{\nu-1}e^{-s}.$$

The integral is $\Gamma(\nu)$ and

$$\sum_{k=0}^\infty \frac{(-1)^k}{(k+1)^\nu} = (1-2^{1-\nu})\zeta(\nu),$$

gives the result.