PROOF OF FORMULA 3.411.30

$$\int_{-\infty}^{\infty} \frac{e^{px} - e^{qx}}{1 - e^{rx}} \frac{dx}{x} = \ln \left[\sin \frac{\pi p}{r} \text{ cosec } \frac{\pi q}{r} \right]$$

Denote the integral by $I(p)$ and differentiate with respect to p to obtain

$$I'(p) = \int_{-\infty}^{\infty} \frac{e^{px} \, dx}{1 - e^{-rx}} = \frac{1}{r} \int_{-\infty}^{\infty} \frac{e^{-pt/r} \, dt}{1 - e^{-t}}$$

after the change of variables $t = -rx$. This last (singular) integral appears as entry 3.311.8 and it yields

$$I'(p) = \frac{\pi}{r} \cot \frac{\pi p}{r}.$$

Integrate back and use $I(q) = 0$ to obtain the result.