PROOF OF FORMULA 3.411.4

$$\int_0^\infty \frac{x^{2n-1}\,dx}{e^{ax}+1} = (1-2^{1-2n})\left(\frac{2\pi}{a}\right)^{2n}\frac{|B_{2n}|}{4n}$$

Formula 3.411.3 states that

$$\int_0^\infty \frac{x^{\nu-1}\,dx}{e^{ax}+1} = \frac{1-2^{1-2n}}{a^{2n}}\Gamma(2n)\zeta(2n).$$

Replace $\nu = 2n$ and use

$$\zeta(2n) = \frac{|B_{2n}|(2\pi)^{2n}}{2(2n)!}$$

to simplify the answer.