PROOF OF FORMULA 3.411.5

$$\int_0^{\ln 2} \frac{x\,dx}{1-e^{-x}} = \frac{\pi^2}{12}$$

Expand the integrand to obtain

$$\int_0^{\ln 2} \frac{x\,dx}{1-e^{-x}} = \sum_{k=0}^{\infty} \int_0^{\ln 2} x e^{-kx}\,dx.$$

Integration by parts shows that

$$\int_0^{\ln 2} x e^{-kx}\,dx = -\frac{\ln 2}{k2^k} + \frac{1-2^{-k}}{k^2}.$$

Therefore

$$\int_0^{\ln 2} \frac{x\,dx}{1-e^{-x}} = \frac{1}{2}\ln^2 2 - \ln 2 \sum_{k=1}^{\infty} \frac{1}{k2^k} + \frac{\pi^2}{6} - \sum_{k=1}^{\infty} \frac{1}{k^2 2^k},$$

where the first term comes from the index $k=0$.

The value

$$\sum_{k=1}^{\infty} \frac{1}{k2^k} = \ln 2$$

comes from the series for $\ln(1-x)$ at $x = 1/2$. The last series is a special value of the *polylogarithm function*

$$\text{PolyLog}[m,x] := \sum_{k=1}^{\infty} \frac{x^k}{k^m}.$$

The evaluation of the integral comes from

$$\text{PolyLog}[2, \tfrac{1}{2}] = \frac{1}{12}\left(\pi^2 - 6\ln^2 2\right).$$

An elementary proof of this evaluation is possible. The details will appear in a future paper.