

PROOF OF FORMULA 3.411.6

$$\int_0^{\infty} \frac{x^{\nu-1} e^{-\mu x} dx}{1 - b e^{-x}} = \Gamma(\nu) \sum_{n=0}^{\infty} \frac{b^n}{(\mu + n)^{\nu}}$$

Expand the integrand to obtain

$$\int_0^{\infty} \frac{x^{\nu-1} e^{-\mu x} dx}{1 - b e^{-x}} = \sum_{n=0}^{\infty} b^n \int_0^{\infty} x^{\nu-1} e^{-(\mu+n)x} dx.$$

The change of variables $t = (\mu + n)x$ yields

$$\int_0^{\infty} \frac{x^{\nu-1} e^{-\mu x} dx}{1 - b e^{-x}} = \sum_{n=0}^{\infty} \frac{b^n}{(\mu + n)^{\nu}} \int_0^{\infty} t^{\nu-1} e^{-t} dt.$$

The integral is $\Gamma(\nu)$ and the result follows.