PROOF OF FORMULA 3.411.8

$$\int_0^\infty \frac{x^{n-1}e^{-px}\,dx}{1+e^x} = (n-1)!\sum_{k=1}^\infty \frac{(-1)^{k-1}}{(k+p)^n}$$

The integral is

$$\int_0^\infty \frac{x^{n-1}e^{-px}\,dx}{1+e^x} = \int_0^\infty \frac{x^{n-1}e^{-(p+1)x}\,dx}{1+e^{-x}}$$

and expanding the integrand in a geometric series

$$\int_0^\infty \frac{x^{n-1}e^{-(p+1)x}\,dx}{1+e^{-x}} = \sum_{k=0}^\infty (-1)^k \int_0^\infty x^{n-1}e^{-(p+1+k)x}.$$

The change of variables $u = (p+1+k)x$ and

$$\int_0^\infty u^{n-1}e^{-u}\,du = \Gamma(n) = (n-1)!,$$

give the result.