PROOF OF FORMULA 3.427.2

$$\int_0^\infty \left(\frac{1}{1-e^{-x}} - \frac{1}{x} \right) e^{-x}\, dx = \gamma$$

Entry 3.427.1 states that

$$\int_0^\infty \left[\frac{e^{-x}}{x} + \frac{e^{-ax}}{e^{-x}-1} \right] dx = \psi(a)$$

Now let $a = 1$ and use $\psi(1) = -\gamma$.