

PROOF OF FORMULA 3.433

$$\int_0^{\infty} x^{p-1} \left[e^{-x} + \sum_{k=1}^n (-1)^k \frac{x^{k-1}}{(k-1)!} \right] dx = \Gamma(p)$$

Define

$$I_n(p) := \int_0^{\infty} x^{p-1} \left[e^{-x} + \sum_{k=1}^n (-1)^k \frac{x^{k-1}}{(k-1)!} \right] dx.$$

Integrate by parts to obtain

$$\begin{aligned} I_n(p) &= - \int_0^{\infty} \frac{x^p}{p} \left[-e^{-x} + \sum_{k=2}^n (-1)^k \frac{x^{k-2} (k-1)}{(k-1)!} \right] dx \\ &= \frac{1}{p} \int_0^{\infty} x^p \left[e^{-x} - \sum_{k=1}^{n-1} (-1)^k \frac{x^{k-1}}{(k-1)!} \right] dx \\ &= \frac{1}{p} I_{n-1}(p+1). \end{aligned}$$

The result now follows by induction of n .