

PROOF OF FORMULA 3.435.2

$$\int_0^\infty \frac{1 - e^{-\mu x}}{x(x+b)} dx = \frac{1}{b} [\ln(b\mu) + \gamma - e^{b\mu} \text{Ei}(-b\mu)]$$

The change of variables $x = bt$ yields

$$\int_0^\infty \frac{1 - e^{-\mu x}}{x(x+b)} dx = \frac{1}{b} \int_0^\infty \frac{1 - e^{-at}}{t(1+t)} dt$$

with $a = b\mu$. This can be written as

$$\int_0^\infty \frac{1 - e^{-at}}{t(1+t)} dt = \int_0^\infty \left(\frac{1}{1+t} - e^{-t} \right) \frac{dt}{t} + \int_0^\infty \left(\frac{e^{-t}}{t} - \frac{e^{-at}}{t(1+t)} \right) dt.$$

Formula 3.435.3 states that the first integral is γ . Using the partial fraction decomposition

$$\frac{1}{t(1+t)} = \frac{1}{t} - \frac{1}{1+t}$$

we obtain

$$\int_0^\infty \frac{1 - e^{-at}}{t(1+t)} dt = \gamma + \int_0^\infty \frac{e^{-t} - e^{-at}}{t} dt + \int_0^\infty \frac{e^{-at}}{1+t} dt.$$

Formula 3.434.2 states that the first integral is $\ln a$. The change of variables $s = (1+t)/a$ shows that the last integral is $-e^a \text{Ei}(-a)$.