PROOF OF FORMULA 3.457.2

$$\int_{-\infty}^{\infty} \frac{xe^x\,dx}{(a+e^x)^{n+3/2}} = \frac{2}{(2n+1)a^{n+1/2}}\left[\ln(4a) - \gamma - 2\psi(2n) + \psi(n)\right]$$

The change of variable $t = e^x$ gives

$$\int_{-\infty}^{\infty} \frac{xe^x\,dx}{(a+e^x)^{n+3/2}} = \int_0^{\infty} \frac{\ln t\,dt}{(a+t)^{n+3/2}}.$$

Define

$$f(b) = \int_0^{\infty} \frac{t^b\,dt}{(a+t)^m}.$$

Let $t = as$ to produce

$$f(b) = a^{b+1-m}\int_0^{\infty} \frac{s^b\,ds}{(1+s)^m} = a^{b+1-m}B(b+1, m-b-1) = a^{b+1-m}\frac{\Gamma(b+1)\,\Gamma(m-b-1)}{\Gamma(m)}.$$

Differentiating with respect to b yields

$$f'(b) = \left[\ln a + \psi(b+1) - \psi(m-b-1)\right] f(b).$$

Then

$$f'(0) = \frac{\ln a - \gamma - \psi(m-1)}{(m-1)a^{m-1}}.$$

The requested integral is $f'(0)$ with $m = n+3/2$. The final expression follows from entry 8.365.6:

$$\psi(z+1/2) = 2\psi(2z) - \psi(z) - 2\ln 2.$$