PROOF OF FORMULA 3.461.3

$$\int_0^\infty x^{2n+1} e^{-px^2}\, dx = \frac{n!}{2p^{n+1}}$$

Let $t = \sqrt{p}x$ to obtain

$$\int_0^\infty x^{2n+1} e^{-px^2}\, dx = \frac{1}{p^{n+1}} \int_0^\infty t^{2n+1} e^{-t}\, dt.$$

The change of variables $s = t^2$ gives

$$\int_0^\infty t^{2n+1} e^{-t}\, dt = \frac{1}{2} \int_0^\infty s^n e^{-s}\, ds.$$

The last integral is $\Gamma(n+1) = n!$, and the formula has been established.