PROOF OF FORMULA 3.461.4

$$\int_{-\infty}^{\infty} (x+ia)^{2n} e^{-x^2}\, dx = \frac{n!\,(2n-1)!!\sqrt{\pi}}{2^n} \sum_{k=0}^{n} (-1)^k \frac{(2a)^{2k}}{(2k)!(n-k)!}$$

Expand the binomial and observe that the integrals corresponding to odd powers of x vanish. Therefore

$$\int_{-\infty}^{\infty} (x+ia)^{2n} e^{-x^2}\, dx = 2\sum_{k=0}^{\infty} \binom{2n}{2k} (-1)^k a^{2k} \int_0^{\infty} x^{2n-2k} e^{-x^2}\, dx.$$

The change of variables $t = x^2$ shows that

$$\int_0^{\infty} x^{2n-2k} e^{-x^2}\, dx = \frac{1}{2}\int_0^{\infty} t^{n-k-1/2} e^{-t}\, dt = \frac{1}{2}\Gamma(n-k+\tfrac{1}{2}).$$

The expression 8.339.2:

$$\Gamma(r+\tfrac{1}{2}) = \frac{\sqrt{\pi}}{2^r}(2r-1)!!$$

gives the result. The final expression can be simplified using

$$\frac{(2n-2k-1)!!}{(2n-2k)!} = \frac{1}{2^{n-k}\,(n-k)!} \text{ and } (2n)! = (2n-1)!!2^n n!.$$