

### PROOF OF FORMULA 3.464

$$\int_0^\infty (e^{-\mu x^2} - e^{-\nu x^2}) \frac{dx}{x^2} = \sqrt{\pi}(\sqrt{\nu} - \sqrt{\mu})$$

Fix  $\nu$  and define

$$f(\mu) = \int_0^\infty (e^{-\mu x^2} - e^{-\nu x^2}) \frac{dx}{x^2}.$$

Then

$$f'(\mu) = - \int_0^\infty e^{-\mu x^2} dx = -\frac{\sqrt{\pi}}{2} \mu^{-1/2},$$

and integrating back gives  $f(\mu) = -\sqrt{\pi\mu} + C$ . The constant  $C$  is determined from the boundary condition  $f(\nu) = 0$ .