

PROOF OF FORMULA 3.468.2

$$\int_0^\infty \frac{x e^{-\mu x^2} dx}{\sqrt{a^2 + x^2}} = \frac{1}{2} \sqrt{\frac{\pi}{\mu}} e^{a^2 \mu} (1 - \operatorname{erf}(a\sqrt{\mu}))$$

Let $x = a \tan t$ to produce

$$\int_0^\infty \frac{x e^{-\mu x^2} dx}{\sqrt{a^2 + x^2}} = a e^{\mu a^2} \int_0^{\pi/2} \sec t \tan t e^{-\mu a^2 \sec^2 t} dt.$$

The change of variables $v = \sec t$ followed by $s = \sqrt{\mu} a v$ yield

$$\int_0^\infty \frac{x e^{-\mu x^2} dx}{\sqrt{a^2 + x^2}} = \frac{e^{\mu a^2}}{\sqrt{\mu}} \int_{a\sqrt{\mu}}^\infty e^{-s^2} ds.$$

Now use

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2} ds$$

to obtain the result.