

**PROOF OF FORMULA 3.471.15**

$$\int_0^{\infty} x^{-1/2} e^{-ax-b/x} dx = \sqrt{\frac{\pi}{a}} e^{-2\sqrt{ab}}$$

Let  $t = x^{1/2}$  to obtain

$$\int_0^{\infty} x^{-1/2} e^{-ax-b/x} dx = 2 \int_0^{\infty} e^{-at^2-b/t^2} dt.$$

The change of variables  $s = 1/t$  reduces the value of the integral to 3.472.3:

$$\int_0^{\infty} e^{-a/s^2-bs^2} \frac{ds}{s^2} = \frac{1}{2} \sqrt{\frac{\pi}{2}} e^{-2\sqrt{ab}}$$