

PROOF OF FORMULA 3.473

$$\int_0^{\infty} e^{-x^n} x^{(m+1/2)n-1} dx = \frac{(2m-1)!!}{2^m n} \sqrt{\pi}$$

The change of variables $t = x^n$ gives

$$\int_0^{\infty} e^{-x^n} x^{(m+1/2)n-1} dx = \frac{1}{n} \int_0^{\infty} e^{-t} t^{m-1/2} dt.$$

This last integral is

$$\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^m} (2m-1)!!$$

that gives the result.