PROOF OF FORMULA 3.475.3

$$\int_0^\infty \left(\exp\left(-x^{2^n}\right) - e^{-x}\right) \frac{dx}{x} = (1 - 2^{-n})\gamma$$

Formula 3.476.2 states that

$$\int_0^\infty \left[\exp(-x^p) - \exp(-x^q)\right] \frac{dx}{x} = \frac{p-q}{pq}\gamma.$$

Now put $p = 2^n$ and $q = 1$ to obtain the current integral.