PROOF OF FORMULA 3.478.2

$$\int_0^\infty x^{q-1} \left[1 - \exp(-\mu x^p) \right] \, dx = -\frac{1}{p} \mu^{-q/p} \Gamma \left(\frac{q}{p} \right)$$

The change of variable $t = \mu x^p$ produces

$$\int_0^\infty x^{q-1} \left[1 - \exp(-\mu x^p) \right] \, dx = \frac{1}{p \mu^{q/p}} \int_0^\infty t^{q/p-1} (1 - e^{-t}) \, dt.$$

Integrating by parts and checking the vanishing of the boundary terms yields

$$\int_0^\infty t^{q/p-1} (1 - e^{-t}) \, dt = -\frac{p}{q} \int_0^\infty t^{q/p} e^{-t} \, dt.$$

The last integral is reconginzed as a gamma value. The identity $\Gamma(x+1) = x\Gamma(x)$ is then used to simplify the result.