PROOF OF FORMULA 3.481.2

$$\int_{-\infty}^{\infty} xe^x \exp(-\mu e^{2x})\, dx = -\frac{\gamma + \ln 4\mu}{4}\sqrt{\frac{\pi}{\mu}}$$

The gamma function is defined by the integral

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x}\, dx.$$

Let $x = t^b$ to obtain

$$\int_0^\infty t^c e^{-t^b}\, dt = \frac{1}{b}\Gamma\left(\frac{c+1}{b}\right)$$

with $c = ab - 1$. A similar scaling yields

$$\int_0^\infty x^c e^{-sx^b}\, dx = \frac{\Gamma(a)}{bs^a}$$

with the same relation of parameters: $c = ab - 1$. Differentiate with respect to c, keeping in mind that $a = (c+1)/b$ gives

$$\int_0^\infty x^c e^{-sx^b} \ln x\, dx = \frac{\Gamma(a)}{b^2 s^a}\left[\psi(a) - \ln s\right].$$

The change of variables $x = e^t$ gives

$$\int_{-\infty}^{\infty} te^{ct} \exp(-se^{bt})\, dt = \frac{\Gamma(c/b)}{b^2 s^{c/b}}\left(\psi(c/b) - \ln s\right).$$

The special case $b = 2$ and $c = 1$ gives the current integral.