PROOF OF FORMULA 3.484

$$\int_0^\infty \left[\left(1 + \frac{a}{qx}\right)^{qx} - \left(1 + \frac{a}{px}\right)^{px} \right] \frac{dx}{x} = (e^a - 1) \ln\left(\frac{q}{p}\right)$$

This is an example of Frullani type

$$\int_0^\infty [f(qx) - f(px)] \frac{dx}{x} = [f(\infty) - f(0)] \ln\left(\frac{p}{q}\right).$$

The function is $f(x) = (1 + a/x)^x$ with $f(0) = 1$ and $f(\infty) = e^a$. This gives the result.