PROOF OF FORMULA 3.513.1

$$\int_0^\infty \frac{dx}{a+b\sinh x} = \frac{1}{\sqrt{a^2+b^2}} \ln\left[\frac{a+b+\sqrt{a^2+b^2}}{a+b-\sqrt{a^2+b^2}}\right]$$

The change of variables $u = e^x$ gives

$$\int_0^\infty \frac{dx}{a+b\sinh x} = 2\int_1^\infty \frac{du}{bu^2+2au-b}.$$

Define

$$u_\pm = \frac{-a \pm \sqrt{a^2+b^2}}{b}$$

and the value of the integral comes from the partial fraction decomposition

$$\frac{1}{bu^2+2au-b} = \frac{1}{\sqrt{a^2+b^2}}\left(\frac{1}{u-u_+} - \frac{1}{u-u_-}\right).$$