PROOF OF FORMULA 3.513.3

$$\int_0^\infty \frac{dx}{a\sinh x + b\cosh x} = \begin{cases} \frac{2}{\sqrt{b^2-a^2}} \tan^{-1}\sqrt{\frac{b-a}{b+a}} & \text{if } b^2 > a^2 \\ \frac{1}{\sqrt{a^2-b^2}} \ln\left[\frac{a+b+\sqrt{a^2-b^2}}{a+b-\sqrt{a^2-b^2}}\right] & \text{if } b^2 < a^2 \end{cases}$$

Let $t = e^{-x}$ to obtain

$$\int_0^\infty \frac{dx}{a\sinh x + b\cosh x} = \frac{2}{b-a}\int_0^1 \frac{dt}{t^2+s},$$

where $s = (b+a)/(b-a)$. The calculation is now divided according to the sign of s. The details are elementary.