PROOF OF FORMULA 3.521.4

$$\int_1^\infty \frac{dx}{x\,\cosh ax} = 2\sum_{k=0}^\infty (-1)^{k+1} \text{Ei}\left(-(2k+1)a\right)$$

Write the integral as

$$\int_1^\infty \frac{dx}{x\,\cosh ax} = 2\int_1^\infty \frac{e^{-ax}}{x} \frac{dx}{1+e^{-2ax}}.$$

Expand the integrand as a geometric series to produce

$$\begin{aligned}
\int_1^\infty \frac{dx}{x\,\sinh ax} &= 2\sum_{k=0}^\infty (-1)^k \int_1^\infty \frac{e^{-(2k+1)ax}}{x}\,dx \\
&= 2\sum_{k=0}^\infty (-1)^k \int_{(2k+1)a}^\infty \frac{e^{-t}}{t}\,dt.
\end{aligned}$$

The result now follows from the definition of the exponential integral

$$\text{Ei}(x) = -\int_{-x}^\infty \frac{e^{-t}}{t}\,dt.$$