

PROOF OF FORMULA 3.527.11

$$\int_0^\infty \frac{x \sinh ax}{(\cosh x)^{2\mu+1}} dx = \frac{\sqrt{\pi}}{4\mu a^2} \frac{\Gamma(\mu)}{\Gamma(\mu + \frac{1}{2})}$$

Differentiate entry 3.512.1

$$\int_0^\infty \frac{dx}{(\cosh ax)^{2\mu}} = \frac{4^{\mu-1}}{a} B(\mu, \mu)$$

with respect to μ . This yields

$$\int_0^\infty \frac{x \sinh ax}{(\cosh x)^{2\mu+1}} dx = \frac{2^{2\mu-2} B(\mu, \mu)}{\mu a^2}.$$

To obtain the form of the answer given here, use the duplication formula for the gamma function

$$\Gamma(2\mu) = \frac{2^{2\mu-1}}{\sqrt{\pi}} \Gamma(\mu) \Gamma(\mu + \frac{1}{2})$$

to establish

$$B(\mu, \mu) = 2^{1-2\mu} B(\frac{1}{2}, \mu).$$

Then replace.