PROOF OF FORMULA 3.546.2

$$\int_0^\infty e^{-bx^2}\,\cosh ax\,dx = \frac{\sqrt{\pi}}{2\sqrt{b}}\exp\left(\frac{a^2}{4b}\right)$$

Let $t = \sqrt{b}x$ to produce

$$\int_0^\infty e^{-bx^2}\,\cosh ax\,dx = \frac{1}{\sqrt{b}}J$$

with

$$J = \int_0^\infty e^{-t^2}\,\cosh ct\,dt$$

and $c = a/\sqrt{b}$. Then write

$$\begin{aligned} J &= \frac{1}{2}e^{c^2/4}\int_0^\infty e^{-(t-c/2)^2}\,dt + \frac{1}{2}e^{c^2/4}\int_0^\infty e^{-(t+c/2)^2}\,dt \\ &= \frac{1}{2}e^{c^2/4}\int_{-c/2}^\infty e^{-u^2}\,du + \frac{1}{2}e^{c^2/4}\int_{c/2}^\infty e^{-u^2}\,du \\ &= e^{c^2/4}\int_{-\infty}^\infty e^{-u^2}\,du \\ &= \frac{\sqrt{\pi}}{2}e^{c^2/4}. \end{aligned}$$

This is the result.