PROOF OF FORMULA 3.546.3

$$\int_0^\infty e^{-bx^2} \sinh^2 ax \, dx = \frac{\sqrt{\pi}}{4\sqrt{b}} \left(\exp\left(\frac{a^2}{b}\right) - 1 \right)$$

Let $t = \sqrt{b}x$ to obtain

$$\int_0^\infty e^{-bx^2} \sinh^2 ax \, dx = \frac{1}{\sqrt{b}} \int_0^\infty e^{-t^2} \sinh^2 ct \, dt.$$

Let J denote the last integral and write $c = a/\sqrt{b}$. Then

$$\begin{aligned}
J &= \frac{1}{4} \int_0^\infty e^{-t^2} \left(e^{2ct} - 2 + e^{-2ct} \right) dt \\
&= \frac{1}{4} \int_0^\infty e^{-t^2+2ct} \, dt - \frac{\sqrt{\pi}}{4} + \frac{1}{4} \int_0^\infty e^{-t^2-2ct} \, dt \\
&= \frac{1}{4} \int_0^\infty e^{-t^2+2ct} \, dt - \frac{\sqrt{\pi}}{4} + \frac{1}{4} \int_{-\infty}^0 e^{-t^2+2ct} \, dt \\
&= \frac{1}{4} e^{c^2} \int_{-\infty}^\infty e^{-(t-c)^2} dt - \frac{\sqrt{\pi}}{4} \\
&= \frac{\sqrt{\pi}}{4} \left(e^{c^2} - 1 \right).
\end{aligned}$$

This is the result.