PROOF OF FORMULA 3.551.6

$$\int_0^\infty \frac{e^{-bx}}{x} \sinh ax \, dx = \frac{1}{2} \ln \frac{b+a}{b-a}$$

Define

$$I(a) := \int_0^\infty \frac{e^{-bx}}{x} \sinh ax \, dx.$$

Then

$$I'(a) = \int_0^\infty e^{-bx} \cosh ax \, dx = \frac{1}{2} \int_0^\infty \left[e^{-(b-a)x} + e^{-(b+a)x} \right].$$

Evaluating these integrals it follows that

$$I'(a) = \frac{1}{2(b-a)} + \frac{1}{2(b+a)}.$$

Integrating with respect to the parameter a yields

$$I(a) = \frac{1}{2} \ln \frac{b+a}{b-a}.$$

The vanishing of the implicit constant of integration comes from letting $a \to 0$.