**PROOF OF FORMULA 3.552.3**

$$\int_0^\infty \frac{x^{\mu-1}e^{-x}}{\cosh x}\,dx = \begin{cases} 2^{1-\mu}(1-2^{1-\mu})\Gamma(\mu)\zeta(\mu) & \mu \neq 1 \\ \ln 2 & \mu = 1 \end{cases}$$

Assume first $\mu \neq 1$. Write the integral as

$$\int_0^\infty \frac{x^{\mu-1}e^{-x}}{\cosh x}\,dx = 2\int_0^\infty \frac{x^{\mu-1}e^{-2x}\,dx}{1+e^{-2x}}.$$

The change of variables $t = 2x$ gives

$$\int_0^\infty \frac{x^{\mu-1}e^{-x}}{\cosh x}\,dx = \frac{1}{2^{\mu-1}}\int_0^\infty \frac{t^{\mu-1}\,dt}{e^t+1}.$$

The entry 9.513.1 states that

$$\zeta(z) = \frac{1}{(1-2^{1-z})\Gamma(z)}\int_0^\infty \frac{t^{z-1}\,dt}{e^t+1}$$

completes the evaluation.

In the case $\mu = 1$ we have

$$\int_0^\infty \frac{e^{-x}\,dx}{\cosh x} = 2\int_0^\infty \frac{e^{-x}\,dx}{e^x+e^{-x}}.$$

The change of variables $t = e^{-x}$ gives

$$\int_0^\infty \frac{e^{-x}\,dx}{\cosh x} = 2\int_0^1 \frac{t\,dt}{t^2+1}.$$

This evaluates as $\ln 2$ directly.