PROOF OF FORMULA 3.552.4

$$\int_0^\infty \frac{x^{2m-1}e^{-ax}}{\cosh ax}\,dx = \frac{1-2^{1-2m}}{2m}|B_{2m}|\left(\frac{\pi}{a}\right)^{2m}$$

The scaling $t = ax$ gives

$$\int_0^\infty \frac{x^{2m-1}e^{-ax}}{\cosh ax}\,dx = \frac{1}{a^{2m}}\int_0^\infty \frac{t^{2m-1}e^{-t}}{\cosh t}\,dt.$$

The integral is evaluated using entry 3.552.3 and the value

$$\zeta(2m) = \frac{2^{2m-1}\pi^{2m}|B_{2m}|}{(2m)!}$$

to obtain the result.