

PROOF OF FORMULA 3.757.2

$$\int_0^{\infty} \frac{\cos(ax)}{\sqrt{x}} dx = \sqrt{\frac{\pi}{2a}}$$

Let $t = ax$ to obtain

$$\int_0^{\infty} \frac{\cos(ax)}{\sqrt{x}} dx = \frac{1}{\sqrt{a}} \int_0^{\infty} \frac{\cos t}{\sqrt{t}} dt.$$

The change of variables $t = u^2$ yields

$$\int_0^{\infty} \frac{\cos t}{\sqrt{t}} dt = 2 \int_0^{\infty} \cos(u^2) du.$$

Formula 3.691.1 states that this integral is $\sqrt{\pi}/2\sqrt{2}$. This establishes the formula.