

PROOF OF FORMULA 3.784.2

$$\int_0^{\infty} \frac{a \sin bx - b \sin ax}{x^2} dx = ab \ln \frac{a}{b}$$

The integral is written as

$$\int_0^{\infty} \frac{a \sin bx - b \sin ax}{x^2} dx = ab \int_0^{\infty} \left(\frac{\sin bx}{bx} - \frac{\sin ax}{ax} \right) \frac{dx}{x}.$$

This integral is now evaluated by Frullani's theorem

$$\int_0^{\infty} (f(bx) - f(ax)) \frac{dx}{x} = [f(0) - f(\infty)] \ln \frac{a}{b}$$

with

$$f(t) = \frac{\sin t}{t}.$$