

PROOF OF FORMULA 4.212.6

$$\int_0^1 \frac{\ln x \, dx}{(a - \ln x)^2} = 1 + (1 + a)e^a \text{Ei}(-a)$$

Let $t = \ln x - a$ to obtain

$$\int_0^1 \frac{\ln x \, dx}{(a - \ln x)^2} = ae^a \int_{-\infty}^{-a} \frac{e^t}{t^2} dt + e^a \int_{-\infty}^{-a} \frac{e^t}{t} dt.$$

The first integral comes from 2.325.2:

$$\int \frac{e^{ax}}{x^2} dx = -\frac{e^{ax}}{x} + a \text{Ei}(ax),$$

and the second one is $\text{Ei}(x)$.