PROOF OF FORMULA 4.215.4

$$\int_0^1 \frac{dx}{\sqrt{\ln \frac{1}{x}}} = \sqrt{\pi}$$

Let $t = \ln \frac{1}{x}$ to produce

$$\int_0^1 \left(\ln \frac{1}{x}\right)^{\mu-1} dx = \int_0^\infty t^{\mu-1} e^{-t}\, dt.$$

This is the standard integral representation of the gamma function, thus

$$\int_0^1 \left(\ln \frac{1}{x}\right)^{\mu-1} dx = \Gamma(\mu).$$

Now put $\mu = \frac{1}{2}$ to obtain the result.