

PROOF OF FORMULA 4.224.6

$$\int_0^{\pi/2} \ln \cos x \, dx = -\frac{\pi}{2} \ln 2$$

The change of variables $x \mapsto \frac{\pi}{2} - x$ gives

$$\int_0^{\pi/2} \ln \cos x \, dx = \int_0^{\pi/2} \ln \sin x \, dx.$$

Therefore

$$\begin{aligned} 2 \int_0^{\pi/2} \ln \cos x \, dx &= \int_0^{\pi/2} \ln \sin x \cos x \, dx \\ &= \int_0^{\pi/2} \ln \left(\frac{\sin 2x}{2} \right) dx \\ &= -\frac{\pi}{2} \ln 2 + \int_0^{\pi/2} \ln \sin 2u \, du \\ &= -\frac{\pi}{2} \ln 2 + \frac{1}{2} \int_0^{\pi} \ln \sin v \, dv \\ &= -\frac{\pi}{2} \ln 2 + \int_0^{\pi/2} \ln \sin u \, du \\ &= -\frac{\pi}{2} \ln 2 + \int_0^{\pi/2} \ln \cos u \, du \end{aligned}$$

This gives the result.