

PROOF OF FORMULA 4.225.1

$$\int_0^{\pi/4} \ln(\cos x - \sin x) dx = -\frac{\pi}{8} \ln 2 - \frac{G}{2}$$

Write $\cos x - \sin x = \sqrt{2} \cos(\pi/4 + x)$. Then

$$\int_0^{\pi/4} \ln(\cos x - \sin x) dx = \frac{\pi}{8} \ln 2 + \int_{\pi/4}^{\pi/2} \ln \cos x dx.$$

The last integral is

$$\int_0^{\pi/2} \ln \cos x dx - \int_0^{\pi/4} \ln \cos x dx = \left(-\frac{\pi}{2} \ln 2\right) - \left(-\frac{\pi}{4} \ln 2 + \frac{G}{2}\right)$$

according to entries 4.224.6 and 4.224.5, respectively.