# PROOF OF FORMULA 4.226.6

$$\int_0^{\pi/2} \ln\left(a^2\cos^2 x + b^2 \sin^2 x\right)\, dx = \frac{1}{2}\int_0^{\pi} \ln\left(a^2\cos^2 x + b^2 \sin^2 x\right)\, dx = \pi \ln\left(\frac{a+b}{2}\right)$$

The integral satisfies the first order partial differential equation

$$a\frac{\partial f}{\partial a} + b\frac{\partial f}{\partial b} = \pi.$$

It follows that $f(a,b) = h(t)$, with $t = a+b$. Repalcing in the differential equation gives $th'(t) = \pi$. Therefore, $h(t) = \pi \ln t + C$. The constant of integration is found to be $-\pi \ln 2$ from entry 4.224.6

$$\int_0^{\pi/2} \ln \cos x \, dx = -\frac{\pi}{2}\ln 2.$$

This completes the proof.