PROOF OF FORMULA 4.232.2

$$\int_0^\infty \frac{\ln x \, dx}{(x+a)(x+b)} = \frac{\ln^2 a - \ln^2 b}{2(a-b)}$$

A partial fraction decomposition of the integrand gives

$$\int_0^\infty \frac{\ln x \, dx}{(x+a)(x+b)} = \frac{1}{2(a-b)} \int_0^\infty \ln x \left(\frac{1}{x+b} - \frac{1}{x+a} \right) dx.$$

The change of variable $x = ct$ gives

$$\int_0^N \frac{\ln x \, dx}{x+c} = \ln c \int_1^{N/c} \frac{dt}{1+t} + \int_1^{N/c} \frac{\ln t \, dt}{1+t}.$$

This yields

$$\int_0^N \frac{\ln x \, dx}{(x+a)(x+b)} = \frac{1}{2(a-b)} \left(\ln b \ln(1+N/b) - \ln a \ln(1+N/a) + \int_{N/a}^{N/b} \frac{\ln t \, dt}{1+t} \right).$$

Now let $N \to \infty$ to produce the result.